Assignment 2

This homework is due Friday Jan 31.

There are total 25 points in this assignment. 22 points is considered 100%. If you go over 22 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should contain full proofs. Bare answers will not earn you much.

1. Method of finite differences

(1) [3pt] Using method of finite differences, find a formula for the sum $1^3 + 2^3 + \cdots + n^3$.

(2) [4pt] Given a fixed integer $m \geq 3$, so-called m-gonal numbers are those that can be organized into an m-gon in a specific way (for example, see a figure for pentagonal numbers in Problem 2.1.10 in textbook). For a given m, m-gonal numbers $p_1^m, p_2^m, \ldots$ can be inductively defined as $p_1^m = 1$, $p_n^m = p_{n-1}^m + (m-2)n + 1$ for $n \geq 2$.

Using method of finite differences, express n-th m-gonal number p_n^m through m and n.

Update. Caution: the above is not a correct definition of polygonal numbers. The correct description is, $p_1^m = 1$, $p_{n+1}^m = p_n^m + (m-2)n + 1$ for $n \geq 1$. The problem, however, still can be solved as stated, which is done in solutions.

2. Division algorithm

(3) [2pt] (2.2.2) Show that any integer of the form $6k+5$ is also of the form $3j+2$, but not conversely.

(4) [2pt] (2.2.6) Show that cube of any integer is of the form $7k$ or $7k \pm 1$.

(5) [3pt] Show that in every Pythagorean triple, at least one number is divisible by 3. (Pythagorean triple is three integers a, b, c such that $a^2 + b^2 = c^2$.)

3. Base b positional number systems

(6) [2pt] Express 2013_4 in ternary.

(7) [2pt] Compose base 5 multiplication table.

(8) (a) [4pt] Explain the following algorithm to multiply two positive integers (i.e. explain why it always works).

To multiply two numbers, form two columns with these numbers in the top entries. For each new row, halve the number in the left column, always rounding down, and double the number in the right column. When the left number reaches 1, strike out all rows where the left number is even. To get the answer, sum up the remaining numbers in the right column. For example, to multiply 75×221, write

75	221
37	442
~~18~~	~~884~~
9	1768
~~4~~	~~3536~~
~~2~~	~~7072~~
1	14144
	16575

Comment. This algorithm is called Peasant Multiplication and was used in a number of ancient cultures when "normal" long multiplication was not accessible. For that there could be two reasons: (1) either positional system was not adopted by the culture; or (2) those who were doing the multiplication lacked education to know the multiplication table — hence the name of this algorithm.

(b) [3pt] Will the same work if we divide and multiply by 3 instead of halving and doubling (and strike out numbers divisible by 3 rather than even ones)? If yes, explain why; if not, give an example of two numbers for which it fails and suggest an adjustment to the algorithm that would make it work.